

# Parity violating observables in radiative neutrino pair emission from metastable atoms <sup>§</sup>

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## ABSTRACT

We report on a possibility of measuring parity violating effects in radiative neutrino pair emission from metastable atoms; asymmetric angular distribution of emitted photons from oriented atoms and emergent circular polarization. Their observation, along with the continuous photon energy spectrum which has 6 thresholds, may be interpreted as events being a combined weak and QED process, emission of  $\gamma\nu_i\nu_j$  in the final state. The method may greatly help to perform neutrino mass spectroscopy using atoms, a systematic determination of the neutrino mass matrix.

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**Introduction** Despite exciting experimental developments in neutrino physics the neutrino is still a very mysterious particle. Do neutrinos obey the same type of Dirac equation as all other charged leptons and quarks, or do they obey two-component Majorana equation in which particle is indistinguishable from its antiparticle? How small is the lightest neutrino mass? These questions are linked to the cosmological scenario of leptogenesis [1] which ultimately aims at a deep understanding of the matter-antimatter imbalance in our universe.

Radiative neutrino pair emission (RNPE) [2], [3]  $|i\rangle_A \rightarrow |f\rangle_A + \gamma + \nu_i \nu_j$  ( $\nu_i$  being the  $i$ -th mass eigenstate ordered in mass by  $0 < m_1 < m_2 < m_3$ ) from a metastable atomic state  $|i\rangle_A$ , a process yet to be discovered, gives the opportunity of systematically measuring the neutrino mass matrix (described by 9 real parameters) and the mass type, Majorana vs Dirac. In the present work we calculate the magnitude of observables that evidently indicates parity violation, providing, along with a missing energy suggestive of invisible neutrino emission, a fundamental tool of unambiguous detection of the weak RNPE process. The RNPE process furthermore gives critical information on three neutrino masses and the unknown mixing angle  $\theta_{13}$ . This way one may advance a step towards verifying important ingredients of leptogenesis theory.

The first experimental discovery of parity violation was made by measuring emitted electron correlation with polarization axis of target in the  $\beta$  and  $\mu$  decay [4]. The measured electron in the  $\beta$  and  $\mu$  decay is replaced here in RNPE by the photon. In RNPE the circular polarization of emitted photon is another good quantity for verification of parity violation. Mass thresholds of invisible neutrino pair emission of mass eigenstates  $\nu_i \nu_j$  are measurable in atomic experiments by a spectrometer with good energy resolution, while in  $\beta$  and  $\mu$  decays only flavor eigenstates  $\nu_e, \nu_\mu$  are observable with poor energy resolution.

In order to increase feasibility of actual experiments one needs enhancement of RNPE rates, for which we use macro-coherence of target atoms [3]. Measurement of parity violation helps much to reduce QED backgrounds even for smaller rates, which thus becomes important together with other means of the background rejection [5].

**Prepared states** In discussions that follow it is essential to have the momentum conservation  $\vec{k} + \vec{p}_1 + \vec{p}_2 = 0$  among 3 emitted light particles,  $\vec{k}$  the momentum of photon,  $\vec{p}_i$  those of neutrinos, taking infinitely heavy atom, along with the usual energy conservation. This way the macro-coherent decay is much similar, in kinematics, to the ordinary decay of a single relativistic particle of mass  $\Delta$  where  $\Delta = E_i - E_f$  is the energy difference of initial and final atomic states. For the momentum-conserving three-body decay of RNPE the energy spectrum of photon has 6 different thresholds which occur at photon energy of  $\Delta/2 - (m_i + m_j)^2/(2\Delta)$ . The threshold locations thus determine neutrino pair masses  $m_i + m_j$ . It turns out that the threshold rise of single photon spectrum is sensitive to neutrino mixing angles such as the unknown  $\theta_{13}$  (see below for more details). How the momentum conservation holds for macro-coherent phenomena [3] shall be explained later.

Atoms we consider belong to a class of  $\Lambda$ -type three level system,  $|i\rangle_A, |n\rangle_A, |f\rangle_A$  with energy relation  $E_n > E_i > E_f$ . RNPE occurs in second order perturbation, a combination of weak and QED process, and has the transition operator of the structure [2],  $G_F \vec{d} \cdot \vec{E} \vec{S}_e \cdot \vec{j}_\nu / (\Delta_{nf} - \omega)$ , where the electron spin operator  $\vec{S}_e$  arises from the neutrino pair emission vertex for atomic transition  $|i\rangle_A \rightarrow |n\rangle_A$ , multiplied by the neutrino current  $\vec{j}_\nu$ , a bilinear form of two plane wave neutrinos, and  $\vec{d} \cdot \vec{E}$  for  $|n\rangle_A \rightarrow |f\rangle_A$  is the usual QED E1 (or M1  $\vec{\mu} \cdot \vec{B}$ ) vertex. Here  $\Delta_{nf} = E_n - E_f$  is the energy difference between two atomic levels, and  $\omega$  is the photon energy. We may classify the weak vertex as Gamow-Teller (GT) type, since it involves the electron spin operator. The Fermi type operator appears as a small relativistic correction, and it usually gives much smaller rates than GT,

GT/Fermi interference being of order  $10^{-4}$  less usually. The transition operator for  $\nu_i\nu_j$  emission is multiplied by  $\xi_{ij} = U_{ei}^*U_{ej} - \delta_{ij}/2$ , with  $U_{ei}$  the neutrino mixing matrix [6]. For simplicity we assume that the Majorana phases are absent, thus  $U_{e1} = c_{12}c_{13}$ ,  $U_{e2} = s_{12}c_{13}$ ,  $U_{e3} = s_{13}$ , with  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$  ([7] for an effect of Majorana phases).

Two energy differences of atomic levels,  $\Delta_{if} = E_i - E_f \equiv \Delta$  and  $\Delta_{ni} = E_n - E_i$ , are involved in the transition matrix element of RNPE; the larger  $\Delta$  and the smaller  $\Delta_{ni}$ , the larger the rate is. Low lying excited levels of rare gas atoms are among the best candidate targets. For instance, an example of Xe gives a good candidate of states;  $|i\rangle_A = 5p^5(^2P_{3/2})6s^2[3/2]_2$  of total electron angular momentum  $J = 2$ , metastable with lifetime O[40] sec,  $|f\rangle_A = 5p^6\ ^1S_0$  the ground state of Xe atom, and  $|n\rangle_A = 5p^5(^2P_{3/2})6s^2[3/2]_1$  of  $J = 1$ . These low lying excited states are well described by two "particle" picture of electron and hole;  $(6s)(5p)^+$ . The system is nearly degenerate,  $\Delta_{ni} \ll \Delta$ , giving a large RNPE rate. In isolated environment  $\Delta_{ni} \sim 0.12$  eV and  $\Delta \sim 8.4$  eV for Xe.

Preparation of atomic polarization in excited states is not difficult in optical experiments using atoms, made possible by tuning excitation laser frequency, if a magnetic field (its direction taken to  $\parallel$  z-axis) is applied to distinguish Zeeman sublevels. Both for Xe atoms of even (having no hyperfine split levels) and odd nuclei (both HFS and Zeeman splitting present) we may use the total angular momentum and its component along the magnetic field  $F, M$  to distinguish these energy levels.

A complication that arises to obtain large PV effects is that one needs a nearly equal mixture of states of magnetic quantum numbers  $M$ , different by  $\pm 1$ . This is made evident below in explicit computations. There may be a variety of methods to realize this mixture, and we consider here a simultaneous application of tilted electric field. As is shown in Figure 1 for odd  $^{131}\text{Xe}$  of nuclear spin  $I = 3/2$  [8], the crossing of different hyperfine levels occurs at  $\sim 1.5$  kG for  $|7/2, -1/2\rangle$  and  $|5/2, 1/2\rangle$ , creating degeneracy (we consulted [9] for this computation). Zeeman splitting due to the magnetic field is the first order effect of the field, while the Stark effect due to the electric field is of the second order in field. Presence of the weak electric field removes the degeneracy at the level crossing point, generating a state of nearly equal mixture of states of the form,  $\cos \eta |F, M\rangle + \sin \eta |F - 1, M + 1\rangle$  of  $\tan \eta \sim \pi/4$ . There are actually two nearly degenerate states, this one and the other orthogonal one. The orthogonal one gives PV effects exactly opposite in sign, hence one needs to separate two contributions. This is experimentally possible, for instance, by adiabatically changing the magnetic field strength and choosing either of the two branches of level repulsion. Parity violating effects then appear as  $\propto \cos \eta \sin \eta$ .

Relevant electronic transitions are of the type  $J = 2 \rightarrow J = 1 \rightarrow J = 0$  (note  $\Delta F = \Delta J$ ). Suppose that the magnetic quantum number along z-axis is denoted by  $M$ . We wish to obtain mixture of states,  $M = -1/2$  and  $1/2$  states for finite PV effects. The electron hole in the inner core  $(5p)^+$  of the initial state has  $J = 3/2$  for relevant  $J = 2 \rightarrow 1$  transition. For initial  $M = \pm 1/2$  mixture one may consider a hole in  $p_-$  and  $p_z$  whose wave functions are  $\propto (x - iy)/\sqrt{2}$  and  $\propto z$ . Suppose that a weak Stark field  $E_s$  is applied to a tilted direction in  $(yz)$  plane, with a tilted angle  $\psi$  from z-axis. Second order Stark effects caused by Hamiltonian  $E_s(z \cos \psi + y \sin \psi)$  then leads to an off-diagonal mixing of the type,  $6s(5p_-)^+ \rightarrow (6s)^2 \rightarrow 6s(5p_z)^+$  giving a mixture of states,  $6s(5p_{\pm})^+$  and  $6s(5p_z)^+$ . The matrix element for this is  $\propto \cos \psi \sin \psi$ , thus is maximal at  $\psi = \pi/4$ . In the rest of numerical estimates we assume  $\psi = \pi/4$ .

**RNPE with invisible neutrino pair** For illustration we consider  $^{131}\text{Xe}$  atomic transitions of the type,

$$\left\{ \begin{array}{l} \cos \eta |7/2, -1/2\rangle \\ + \sin \eta |5/2, 1/2\rangle \end{array} \right\} \rightarrow \left\{ \begin{array}{l} |5/2, -3/2\rangle \\ |5/2, -1/2\rangle \\ |5/2, 1/2\rangle \\ |5/2, 3/2\rangle \end{array} \right\} \rightarrow \left\{ \begin{array}{l} |3/2, -3/2\rangle \\ |3/2, -1/2\rangle \\ |3/2, 1/2\rangle \\ |3/2, 3/2\rangle \end{array} \right\}. \quad (1)$$

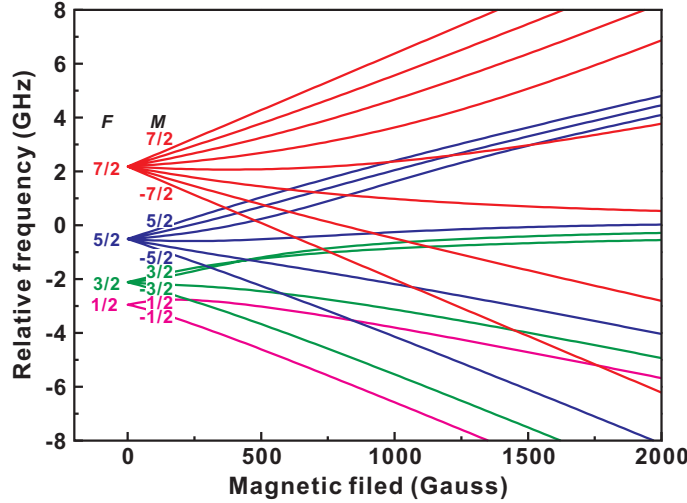


Figure 1: Zeeman levels of odd  $^{131}\text{Xe}$  atom.

Summation over 4 final states is inevitable in a simplest experimental setup that does not measure final atomic states.

Neutrinos are difficult to detect, but one may measure energy, emission angle, and circular polarization of the accompanying photon in RNPE, and derive event rates of specified photon energy-momentum,  $\vec{k}$ , and helicity  $h_\gamma$ , in the form of continuous spectrum. On the other hand, we sum over non-observable variables; neutrino helicities  $h_i$  and their momenta  $\vec{p}_i$  of the pair. The neutrino helicity summation in rates [2] gives bilinear terms of neutrino momenta;  $K_{ij}^S =$

$$\sum_{h_1, h_2} j_M^i (j_M^j)^\dagger = \left(-\frac{m_1 m_2}{E_1 E_2} \delta_M + 1\right) \delta_{ij} + \frac{1}{E_1 E_2} (p_1^i p_2^j + p_1^j p_2^i - \delta_{ij} \vec{p}_1 \cdot \vec{p}_2). \quad (2)$$

The case  $\delta_M = 1$  is applied for Majorana neutrinos, and  $\delta_M = 0$  for Dirac neutrinos. The subsequent neutrino momentum integration (with  $E_i = \sqrt{p_i^2 + m_i^2}$  neutrino energy), with  $\int dP_\nu = \int d^3 p_1 d^3 p_2 \delta^3(\vec{k} + \vec{p}_1 + \vec{p}_2) \delta(\Delta - \omega - E_1 - E_2) K_{ij}^S$ , gives a 1-dim. neutrino energy integral  $\int dE_1$ , which can be done analytically.

The result of neutrino helicity and momentum summation gives quadratic forms of photon momentum,  $k_i k_j$  and  $\delta_{ij} \vec{k}^2$  functions of photon energy  $|\vec{k}| = \omega$ . These quadratic forms are to be multiplied by a product of electron spin operators  $S_i S_j$ . Since an approximate azimuthal symmetry around the magnetic (or atomic polarization) axis exists, one may integrate, or average, over the azimuthal angle  $\varphi$  of emitted photons,  $\int_0^{2\pi} d\varphi$ . This allows omitting various bilinears like  $k_z k_x \propto \cos \varphi$ ,  $k_z k_y \propto \sin \varphi$ ,  $k_x k_y \propto \cos \varphi \sin \varphi$  which all vanish after the azimuthal angular integration. Thus, only two types of effective electron spin operators  $S_\pm S_\mp, S_z S_z$ , ( $S_\pm = S_x \pm i S_y$ ) contribute to rate calculation.

For QED vertex we use two independent circular polarizations of  $h_\gamma = \pm 1$ , and write QED vertex amplitude  $\vec{d} \cdot \vec{E}$  as circularly polarized electric field  $\times \frac{1+c}{2} d_+ \pm \frac{1-c}{2} d_- \pm \frac{s}{\sqrt{2}} d_z$  with  $c = \cos \theta$ ,  $s = \sin \theta$  ( $\theta$  the photon emission angle from polarization axis) and  $d_\pm = \mp(d_x \pm i d_y)/\sqrt{2}$ . The squared amplitude  $d_i d_j E_i E_j^*$  is then bilinear function of the angle factor  $c, s$ .

An example of parity violating (PV) term ending at the final state  $|3/2, -1/2\rangle$  with emission of

$h_\gamma = \pm 1$  photon gives a rate proportional to  $\cos \eta \sin \eta \times$

$$\frac{\pm s(1 \pm c)}{2\sqrt{2}} \langle 5/2, -3/2 | j_+ S_- | 7/2, -1/2 \rangle \langle 5/2, 1/2 | j_- S_+ | 5/2, -1/2 \rangle \\ \times \langle 3/2, -1/2 | E_- d_+ | 5/2, -3/2 \rangle \langle 5/2, -1/2 | E_z^* d_z | 3/2, -1/2 \rangle .$$

This term gives the angular asymmetry  $\propto \sin \theta \cos \theta$  for unpolarized (averaged over helicity  $h_\gamma = \pm 1$ ) photon emission, asymmetric with respect to  $\theta = \pi/2$ . At the same time the same term gives circular polarization, when summed over the helicity weight  $\sum_{h_\gamma} h_\gamma \times \text{rate}$ , leading to angular symmetric terms  $\propto \sin \theta$ .

Thus, unless other added terms cancel in total, two PV effects remain as  $\propto \cos \eta \sin \eta$ .

Atomic wave functions we need for this Xe case are E1 dipole between the intermediate  $J = 1$ , and the final  $J=0$  states, for which we use the measured decay rate  $\gamma_{nf}$  given by  $\omega^2 \sum_{\text{pol}} |\vec{d} \cdot \vec{E}|^2 / \pi = \omega^3 \gamma_{nf} / \Delta_{nf}^3$ . The initial and the intermediate wave function overlap is due to the electron spin flip caused by the neutrino pair emission, hence having a large electronic orbital overlap. This part can be worked out approximately by using Clebsch-Gordon (CG) coefficients [10] alone.

**Spectrum rates and PV observables** A general rate formula may be given under specified photon circular polarization  $h_\gamma = \pm 1$ ,

$$\frac{d^2 \Gamma}{d\omega d \cos \theta}(\omega, h_\gamma) = \frac{G_F^2 \gamma_{nf} n N \omega^3}{8\pi^2 \Delta_{nf}^3 (\Delta_{nf} - \omega)^2} \sum_{ij} |c_{ij}|^2 (b_4 c^4 + b_2 c^2 + b_0 + s(a_3 c^3 + a_1 c) \\ + h_\gamma (e_3 c^3 + e_1 c + s(f_2 c^2 + f_0))) , \quad (3)$$

where  $c = \cos \theta, s = \sin \theta$ . Here  $a_i, b_i, e_i, f_i$  of mass dimension 2 are functions of variables  $\omega$  and elements of the neutrino mass matrix, containing product of 4 CG coefficients. Parity violating effects appear in  $a_i$  and  $f_i$  terms  $\propto \cos \eta \sin \eta$ . Parity conserving terms  $\propto b_i, e_i$  are multiplied by  $\cos^2 \eta$  or  $\sin^2 \eta$ .

Without measurement of circular polarization  $h_\gamma$ , there are components, symmetric one (even function of  $\cos \theta$ ) relative to the atomic polarization axis, and asymmetric one (odd function of  $\cos \theta$ ). The total rate  $\propto b_4/5 + b_2/3 + b_0$  is an integral of symmetric functions. The angular asymmetry gives parity violating effect; for instance, the difference in the forward (F) and the backward (B) hemispheres at some particular angle  $\theta$  and  $\pi - \theta$  of  $0 \leq \theta \leq \pi/2$ . We plot in Figure 2 the angular asymmetry defined by the ratio of FB difference to FB sum,  $s(a_3 c^3 + a_1 c) / (b_4 c^4 + b_2 c^2 + b_0)$ , taking several  $\theta$  of  $\pi/10, \pi/6, \pi/4, \pi/3, 2\pi/5$  (with  $\eta = \pi/4$ ).

With measurement of circular polarization, the simplest P-odd quantity is the relative circular polarization as a function of angle summed at two angles  $\theta$  and  $\pi - \theta$ , given by  $s(f_2 c^2 + f_0) / (b_4 c^4 + b_2 c^2 + b_0)$ . We plot in Figure 3 their spectrum (taking  $\eta = \pi/4$ ) at the same angles as in Figure 2.

Both of these PV quantities are of order unity, typically  $O[0.1]$ , even far away from neutrino mass thresholds, since parity violation is intrinsic to the involved weak interaction not related to the finiteness of neutrino masses. The feature of large parity violation far from thresholds where the event rate is larger can be used for rejection of QED backgrounds and a positive identification of RNPE. The magnitude of PV quantities differ significantly at different emission angles.

Parity-odd quantity thus computed is almost of equal magnitude in the Majorana and the Dirac cases, the difference being  $\leq O[2 \times 10^{-4}]$  in Xe case of a large energy difference  $\sim 8.4$  eV. The Majorana vs Dirac distinction using P-even quantity is made easier by using atomic system of smaller energy difference close to neutrino pair masses such as  $2m_3$  (the largest pair mass); the rate difference may become several  $\times 10^{-2}$  for  $\Delta < 1$  eV.

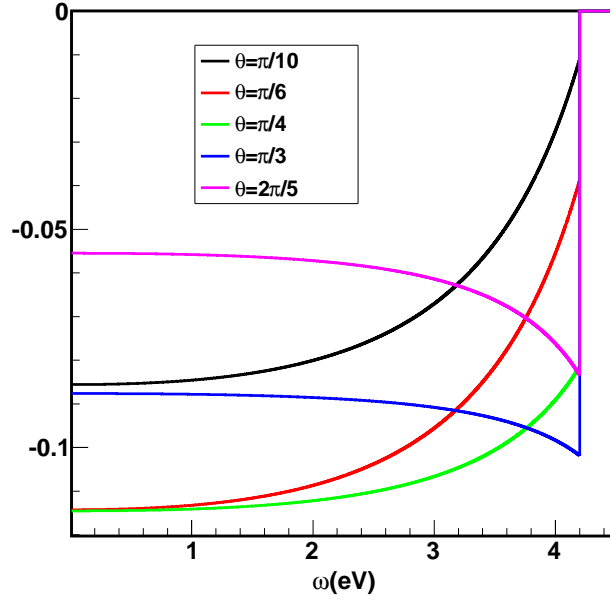


Figure 2: Angular asymmetry of emitted photon at several angles  $\theta$  from oriented  $^{131}\text{Xe}$  atoms in a metastable  $J = 2, F = 7/2$  state as a function of photon energy  $\omega$ . Assumed neutrino masses are 1meV, 10 meV, 50meV, with mixing angles,  $\sin^2 \theta_{12} = 0.3, \sin^2 \theta_{13} = 0.039$  assuming the normal hierarchy [6].

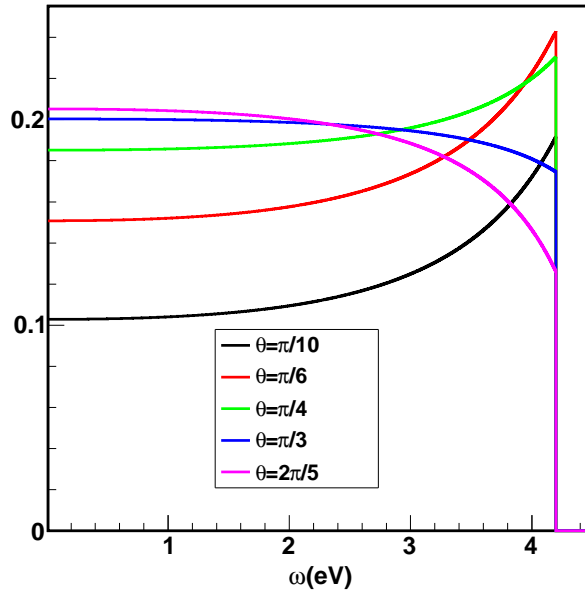


Figure 3: Circular polarization for the Xe metastable state at several angles from oriented atoms. Assumed parameters are the same as in Figure 2.

**Macro-coherent RNPE** Coherence of targets is important, both to enhance the event rate and to determine the threshold location. Ordinarily, the momentum conservation in elementary processes does not play important roles in atomic radiative decay, since atomic recoil is very small and difficult to measure. We however utilize the macro-coherence of target atoms, similar to, but more enhanced in rates than (hence called the macro-coherence), a single photon superradiance [11].

In macro-coherent phenomena more than two light particles such as  $\gamma\gamma$  in two photon emission and  $\gamma\nu_i\nu_j$  in RNPE are involved in the final state of decay product. If the phases of emitted plane-wave particles of product amplitude  $e^{i\sum_i \vec{k}_i \cdot \vec{x}_d}$  from various target sites at  $\vec{x}_d$  (distributed uniformly with a number density  $n$ ) are coherently added, one has the macro-coherent rate in the large target volume ( $V$ ) limit [12],  $\Gamma = nN \int \left( \Pi_i \frac{d^3 k_i}{(2\pi)^3} \right) (2\pi)^4 \delta(\Delta - \sum_i \omega_i) \delta(\sum_i \vec{k}_i) |\mathcal{M}(\vec{k}_i)|^2$ , ( $N = nV$ ), a result of coherence not restricted by the wavelength of emitted photon, quite unlike in the single photon superradiance [11]. Thus, the coherence prevails in a macroscopic target region, and one expects the momentum conservation,  $\vec{k} + \vec{p}_1 + \vec{p}_2 = 0$  for RNPE, and a large enhanced rate.

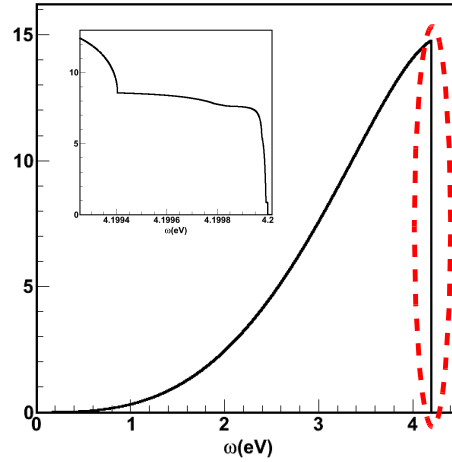


Figure 4: RNPE spectrum rate for the  $^{131}\text{Xe}$  metastable state in the unit of  $nNG_F^2\gamma_{nf}/(2\pi^2\Delta_{nf}^3)$  of eq.(3). The red-dotted threshold region including all six is expanded in the inset to exhibit threshold rises. Assumed parameters are the same as in Figure 2.

Coherence dynamically develops, and the key for this is the seed of quantum fluctuation of fields or successive spontaneous emission within the active target volume, as emphasized in superradiance phenomena [11]. Moreover, an active external trigger by a pulsed laser may help much for the coherence development [13]. Analysis based on the Maxwell-Bloch equation [11] is expected to give strong angular correlation along the axis of exciting and triggering laser, an effect not evident in the present argument and ones in [3] as well.

The total rate  $\int d\omega d\Gamma/d\omega$  from the entire target roughly scales with  $nNG_F^2\gamma_{nf}/(2\pi^2\Delta_{nf}^3)$  times an integrated number. This is numerically  $\sim 4 \times 10^{-9} (n/10^{24}\text{cm}^{-3})(N/10^{20})(\gamma_{nf}/10^9\text{s}^{-1})$  Hz times an integrated spectrum in the  $\text{eV}^4$  unit. A 100 gr of solid  $^{131}\text{Xe}$  gives the total rate of order  $\sim 3 \times 10^{-5}$  Hz, taking into account the enhancement due to the soliton formation crudely estimated in [5] to be  $O[10^4]$ . One needs a more detailed numerical integration of Maxwell-Boltzmann equation to give a more accurately determined rate.

The elementary rate from a single atom may be estimated by taking  $N = 1$ , and replacing the target number density  $n$  by the phase space density  $\sim \Delta^3$  of atomic decay, which is  $\sim \text{wavelength}^{-3}$ . Thus, the macro-coherent rate is enhanced by the factor of the ratio of target volume/wavelength<sup>3</sup> compared to the elementary rate. In contrast the single photon superradiance has the enhancement factor of order, (linear dimension/wavelength)<sup>2</sup> [11].

We show in Figure 4 the total spectrum rate, taking  $\eta = \pi/4$ . A global feature of the macro-coherent spectrum is the sharp rise from thresholds and a fast decrease towards small photon energy region, ultimately  $\propto \omega^3$ . The step around 4.1994 eV is due to (33) mass threshold, whose location, but not spectral shape, is insensitive to the angle factor  $\theta_{13}$  if  $\sin^2 \theta_{13} \leq 0.039$  (the present limit). In order to locate thresholds one needs accuracy of photon energy resolution of order  $10^{-3}$  or even less. This accuracy is not problematic, since one can trigger RNPE by laser whose energy resolution is far better. The fraction that threshold regions occupy is however not large; for instance the ratio of integrated rate of the inset part to the total in Figure 4 is  $\sim 5 \times 10^{-4}$  for all 6 threshold regions. We also worked out the case of inverted hierarchy of neutrino masses; the difference between the two cases is not overwhelming, but a sizable spectral shape difference exists between the two cases, the normal and the inverted hierarchies. In another word one can equally investigate and experimentally distinguish the normal and the inverted cases by this type of atomic experiments.

In this Xe example it is necessary to obtain large high statistics data near threshold regions for the precision neutrino mass spectroscopy. This is due to that Xe level spacing  $\sim 8.4\text{eV}$  is somewhat large compared to the anticipated neutrino mass  $\sim 0.05\text{eV}$  in the assumed normal hierarchy case. Atoms of much smaller energy spacing are far better for the purpose of precision neutrino mass spectroscopy.

To obtain a large enhanced macro-coherent rate, one needs a collection of atomic targets of  $O[\text{Avogadro number}] \text{ cm}^{-3}$ , well isolated among themselves and weakly interacting with environment. An example is a matrix implantation of a fraction of targets within solid matrix such as para- $\text{H}_2$ .

In summary, we discussed two types of parity violating effects; angular asymmetry from oriented atoms and emergent circular polarization, as a means to positively verify weak RNPE process, which can be used to systematically determine the neutrino mass matrix.

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- [12] However, if phases cancel by some other reasons, as in the dilute gas, the incoherent sum results, giving the rate  $N$  (the total target number)  $\times$  the elementary rate for a single atom without the momentum conservation. Thus, on general grounds, one cannot take the dilute gas limit in the formula of macro-coherent rate.
- [13] For instance, N.W. Carlson et al. *OPTICS COMMUNICATIONS*, Volume 32, number 2 (February, 1980).